Reynolds number.[11] The I_{sp} efficiency is the ratio of the actual I_{sp} to the inviscid I_{sp}

$$\eta = \frac{I_{sp_{actual}}}{I_{sp_{ideal}}} \tag{4}$$

and is thus a measure of viscous losses. Inviscid I_{sp} values were calculated with the TDK code.[9] The nozzle throat Reynolds number is defined as

$$Re^* = 2\dot{m}/\pi r^* \mu_c \tag{5}$$

where r^* is the nozzle throat radius, and μ_c is the chamber viscosity. For a generally constant pressure in the nozzle throat, faster kinetics produce higher temperatures and lower densities. The Reynolds number and η values decrease as density decreases, indicating increasing boundary-layer thickness and viscous losses. Table 2 shows that the net I_{sp} actually increases with increasing kinetic rate. These opposite trends indicate that kinetic losses are the dominant loss mechanism. Increasing the recombination rate reduces the kinetic losses, but increases the nozzle viscous losses, thus moderating the I_{sp} improvement.

Because of the low initial degree of dissociation, the uncertainty in kinetic rate only produced a 0.2% I_{sp} uncertainty with T_c = 2780 K and P_c = 345 kPa. At the lowest chamber temperature (2220 K), the I_{sp} uncertainty caused by the kinetic rate is negligible. Halving the chamber pressure from 345 to 173 kPa at 2780 K halved the thrust and mass flow rate, slightly decreased the I_{sp}, and produced a 0.2% I_{sp} uncertainty. Therefore, chamber pressure has only a slight impact on the influence of hydrogen kinetic rate uncertainty.

Based on this investigation and the recommendations of other researchers, the Baulch et al.[6] kinetic rates are recommended as nominal rates for future numerical predictions of solar thermal rocket performance.

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Off-Design Performance Prediction of Single-Spool Turbojets Using Gasdynamics

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Nomenclature

A = area of exhaust nozzle
C_p = specific heat at constant pressure
M = Mach number
m = mass flow
N = engine mechanical speed
P = pressure
R = gas constant
T = temperature
ε_c = compressor pressure ratio
η_c = polytropic compressor efficiency
η_m = mechanical efficiency
η_t = polytropic turbine efficiency
ψ = $T_{t4}/T_{t2}/(T_{t4}/T_{t2})_{des}$

Subscripts
a = air
des = design
t = total
0 = ambient atmosphere

I. Introduction

OFF-DESIGN performance of aeroengines is of major interest, not only to the user, but also the manufacturer, as the analysis can be of use for preliminary performance calculations for parametric studies, particularly for aircraft design purposes.

Mattingly et al.[1] developed a code that did not need the use of performance maps, but it gave results having more than 10–15% error in fuel flow and thrust, even at cruise flight conditions. Wittenberg[2] devised an approach based on gasdynamic relationships, making the use of compressor and turbine maps redundant. The present work is a modified extension of Wittenberg's approach,[2] as it tries to compute overall engine performance by ascertaining turbine entry temperature (TET), knowing just the flight conditions and the corrected engine speed.

II. Off-Design Analysis

The design point analysis of a single-spool turbojet can usually be done by thermodynamic component-to-component analysis, starting with a diffuser, compressor, combustor, turbine, and a propelling nozzle. It is straightforward and can be found in text, such as *Gas Turbine Theory*.[3] Figure 1 shows a schematic view of a single-spool turbojet with station numbering.

The off-design analysis revolves around two conditions: 1) when the aircraft is idling or descending (in both cases the

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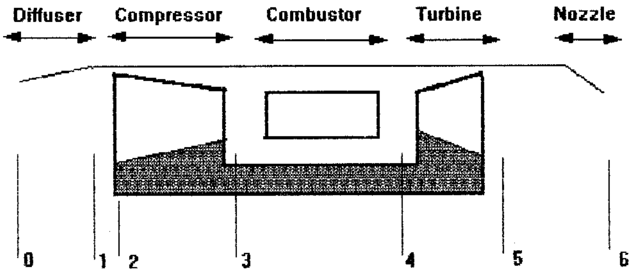

Fig. 1 Schematic view of a single-spool turbojet.

exhaust nozzle is unchoked) and 2) in cruise flight (the nozzle is choked). When studying off-design performance, the independent variables are basically the flight conditions and throttle settings. In this study, the mass flow characteristics of turbines have been generalized to nozzle characteristics; i.e., a single mass flow curve with a constant turbine efficiency has been assumed.

To match the turbine and exhaust nozzle mass flow, we have the following relation:

$$m_g\sqrt{T_{t5}}/P_{t5} = (m_g\sqrt{T_{t4}}/P_{t4})(P_{t4}/P_{t5})^{1-[\eta_t(\gamma_g-1)/2\gamma_g]} \tag{1}$$

where $m_g\sqrt{T_{t4}}/P_{t4}$ is the turbine mass flow. As the mass flows through the nozzle $m_g\sqrt{T_{t5}}/P_{t5} = f(P_{t5}/P_0)$, we can find the turbine pressure ratio for any nozzle pressure ratio P_{t5}/P_0, using Eq. (1). When the propelling nozzle is choked, there is a fixed turbine operating point, which is independent of the nozzle pressure ratio. Moreover, the temperature ratio across the turbine T_{t5}/T_{t4} becomes fixed, and it is the same for both design and off-design conditions. The compatibility of power between the turbine and compressor for a constant C_{pg}, C_{pa}, η_c, and η_m gives the following relation:

$$T_{t3}/T_{t2} - 1 = K_0(T_{t4}/T_{t2})(1 - T_{t5}/T_{t4}) \tag{2}$$

Equation (2) can be further modified to give a new compressor pressure ratio (CPR) at an off-design throttle setting (T_{t4}/T_{t2}), as

$$\varepsilon_c = \{1 + (T_{t4}/T_{t2})/(T_{t4}/T_{t2})_{\text{des}}[(\varepsilon_c)_{\text{des}}^{\gamma_a-1/\eta_c\gamma_a} - 1]\}^{\eta_c\gamma_a/\gamma_a-1} \tag{3}$$

On satisfying the essential condition of compatibility of mass flow between the turbine and compressor, we get a new value of compressor mass flow as

$$m_a\sqrt{T_{t2}}/P_{t2} = (m_a\sqrt{T_{t2}}/P_{t2})_{\text{des}}(\sqrt{T_{t2}/T_{t4}})/(\sqrt{T_{t2}/T_{t4}})_{\text{des}}\varepsilon_c/(\varepsilon_c)_{\text{des}} \tag{4}$$

Once the new values of ε_c and $m_a\sqrt{T_{t2}}/P_{t2}$ are estimated, then other temperatures and pressures for other components can easily be calculated. This procedure gives new values of specific thrust and net specific fuel consumption.

For the unchoked case there is no fixed operating point of the turbine and, hence, T_{t5}/T_{t4} is not constant. An equivalent expression to Eq. (2) can be expressed as

$$1/\psi(T_{t3}/T_{t2} - 1)/(T_{t3}/T_{t2} - 1)_{\text{des}} = (1 - T_{t5}/T_{t4})/[1 - (T_{t5}/T_{t4})_{\text{des}}] \tag{5}$$

Using generalized turbine and nozzle characteristic curves for constant flow areas of turbine and nozzle, we can get for each value of P_{t5}/P_0 a definite value of P_{t5}/P_{t4}, T_{t5}/T_{t4}, and P_{t3}/P_0. The curve between P_{t3}/P_0 and the right-hand side of Eq. (5) is called the turbine/nozzle characteristic curve. Similarly, a compressor curve can be obtained between P_{t3}/P_0 and the left-hand side of Eq. (5) for a particular value of M and temperature ratio ψ. The matching of compressor and turbine/nozzle characteristic curves yields an intersection point value of P_{t3}/P_0. Knowing this, values of CPR and other performance parameters can be calculated.

III. Validation of Model

The purpose of this analysis is to validate the mathematical model, both for choked and unchoked propelling nozzle conditions, with experimental data from the performance reports.[4,5] The Viper 20F-10 is a single-shaft turbojet, and at its design point of International Standard Atmosphere (ISA) + 10°C it develops 13.1 kN (3000 lbf) at takeoff conditions, with a specific fuel consumption of 0.107 kg/h-N (1.05 lb/h-lbf).

The flow areas of the exhaust nozzle and turbine are fixed using the design point parameters such as TET = 1145 K, CPR = 5.15, and air mass flow = 22.32 kg/s at sea level static (SLS) (ISA + 10) conditions. For computing off-design performance, the user has to input TET as well as the flight condition. On assuming a quadratic relationship[6] between TET/T_{t1} and corrected compressor speed $N/\sqrt{T_{t1}}$, we get

$$\text{TET} = A(T_{t1}) + BN\sqrt{T_{t1}} + CN^2 \tag{6}$$

Having determined the three coefficients A, B, and C (Ref.

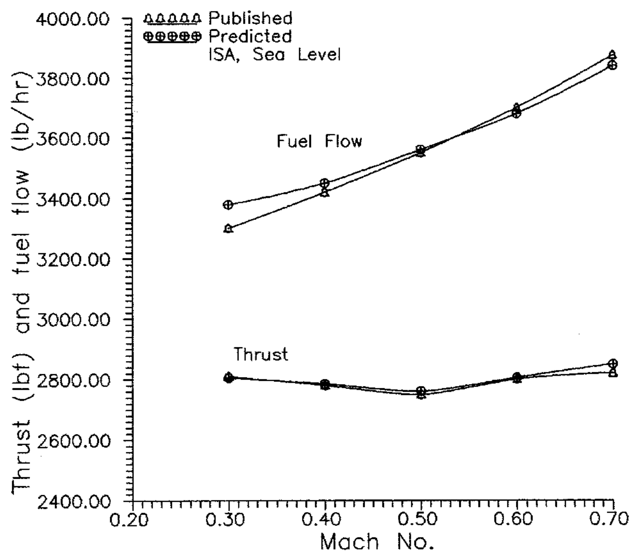

Fig. 2 Comparison of predicted and published performance data for Viper 20F-10 at ISA, sea level.

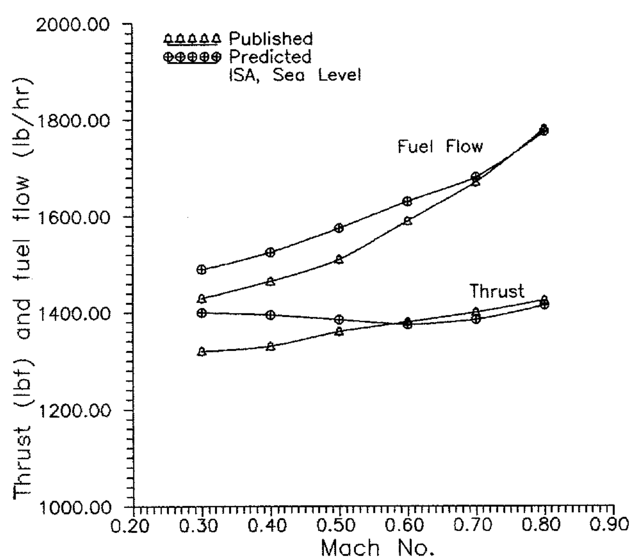

Fig. 3 Comparison of predicted and published performance data for Viper 20F-10 at ISA, 6.1 km.

7), we found that TET is in good agreement with the published data of TET.[5] The code is run at sea-level ISA conditions, 3048, 6096, and 9144 m with varying M. Figures 2 and 3 show the predicted values of thrust and fuel flow at different altitudes as compared with published data.[4,5] It can be seen that even when the component efficiencies (polytropic) are assumed constant throughout, there is a good relationship between predicted and published data.

IV. Conclusions

The following conclusions have been derived:

1) The fact that predictions are in good agreement with experimental data proves the validity of the major assumption that the constraint of a choked exhaust nozzle dictates the aerothermodynamic behavior of all upstream components.

2) For altitudes beyond 8000 m, we need to take compressor efficiency variation into account for better results, because in single-spool turbojets there is more variation of η_c as compared to twin-spool engines.

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